Ashraf & Germina, *Cogent Mathematics* (2016), 3: 1186135
http://dx.doi.org/10.1080/23311835.2016.1186135

cogent mathematics

PURE MATHEMATICS | RESEARCH ARTICLE

Double domination in signed graphs

P.K. Ashraf[1]* and K.A. Germina[2]

Abstract: A graph with either positive or negative labels on the edge becomes a signed graph. Given a signed graph $\Sigma = (V, E, \sigma)$, a subset D of V is said to be a *double dominating set* for Σ, if it satisfies the following conditions: (i) every vertex u of Σ is either in D and u has at least one neighbour in D or whenever $u \in V \setminus D$, $|N(u) \cap D| \geq 2$ (ii) $\Sigma[D:V \setminus D]$ is balanced where $N(u)$ denotes the open neighbourhood of a vertex u and $\Sigma[D:V \setminus D]$ is the subgraph of the Σ induced by the edges between the vertices in D and $V \setminus D$. In this paper, we initiate the discussion on the double domination in signed graphs.

Subjects: Advanced Mathematics; Combinatorics; Discrete Mathematics; Mathematics & Statistics; Science

Keywords: signed graph; domination; double domination

Mathematics subject classifications: 05C10; 05C22

Received: 06 March 2016
Accepted: 21 April 2016

*Corresponding author: P.K. Ashraf, Department of Mathematics, Government Arts and Science College, Kondotty, Malappuram, India
E-mail: ashrafkalanthod@gmail.com

Reviewing editor:
Lishan Liu, Qufu Normal University, China

Additional information is available at the end of the article

1. Introduction

Graphs used in this article, unless otherwise mentioned, will be undirected, simple and finite. For all definitions in (unsigned) graph theory, one can refer to Harary (1972) and for the terminology and definitions in the theory of dominations for simple graphs, we refer the reader to Chartrand and Zang (2009), Haynes, Hedetneimi, and Slater (1998a,1998b). Signed graphs (also called sigraphs) are graphs with positive or negative labels on the edges. Formally, a signed graph is an ordered pair $\Sigma = (G, \sigma)$ where $G = (V, E)$ is a graph called the *underlying graph* of Σ and $\sigma: E \to \{+1, -1\}$ called a *signing*, is a function (also called a *signature*) from the edge set E of G into the set $\{+1, -1\}$. For more details on theory and applications of signed graphs, one may refer to Germina and Shahul Hameed (2010), Germina, Shahul Hameed, and Zaslavsky (2011), Harary (1953), Zaslavsky (1982, 1998).

ABOUT THE AUTHORS

P.K. Ashraf is presently working as an assistant professor in the Department of Mathematics, Government Arts and Science College, Kondotty, Kerala, India. He has more than 4 years of research experience and 10 years of teaching experience. He has published his research findings in many recognized international journals.

K.A. Germina is currently a professor in Mathematics in University Botswana, Gaborone, Botswana. She has more than two decades of teaching experience and 15 years of research experience. She has more than a hundred research publications and she is the reviewer and editor for many journals and premiere reviewing services.

PUBLIC INTEREST STATEMENT

Domination in graphs is one of the major research areas in graph theory. Currently, many interesting and important researches are taking place in this area. Double domination is a particular type of domination and the double domination in graphs is relative new research area and hence there is a wide scope for studies in this particular area of domination theory. Well-known mathematicians F. Harary and T. Haynes initiated the studies in the double domination in graph theory. Later, B. D. Acharya extended the domination theory to different types of signed graphs. Being a relatively new research area, double domination in graphs offers much further investigations. Domination theory has proved to have many applications in many theoretical as well as practical real-life problems like optimization problems, communication problems, network problems, etc.