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ON THE DIMENSION OF VERTEX LABELING OF k-UNIFORM DCSL OF k-UNIFORM CATERPILLAR

A distance compatible set labeling (dcsl) of a connected graph G is an injective set assignment $f : V(G) \to 2^X$, X being a nonempty ground set, such that the corresponding induced function $f^{\oplus} : E(G) \to 2^X \setminus \{\varnothing\}$ given by $f^{\oplus}(uv) = f(u) \oplus f(v)$ satisfies $| f^{\oplus}(uv) | = k^f_{(u,v)} d_G(u,v)$ for every pair of distinct vertices $u, v \in V(G)$, where $d_G(u,v)$ denotes the path distance between u and v and $k^f_{(u,v)}$ is a constant, not necessarily an integer. A dcsl f of G is k-uniform if all the constant of proportionality with respect to f are equal to k, and if G admits such a dcsl then G is called a k-uniform dcsl graph. The *k-uniform dcsl index* of a graph G, denoted by $\delta_k(G)$ is the minimum of the cardinalities of X, as X varies over all k-uniform dcsl-sets of G. A *linear extension* $\mathbf{L}$ of a partial order $\mathbf{P} = (P, \preceq)$ is a linear order on the elements of P, such that $x \preceq y$ in $\mathbf{P}$ implies $x \preceq y$ in $\mathbf{L}$, for all $x, y \in P$. The dimension of a poset $\mathbf{P}$, denoted by $dim(\mathbf{P})$, is the minimum number of linear extensions on $\mathbf{P}$ whose intersection is '$\preceq$'. In this paper we prove that $dim(\mathcal{F}) \leq \delta_k(P_n^{+k})$, where $\mathcal{F}$ is the range of a k-uniform dcsl of the k-uniform caterpillar, denoted by P_n^{+k} $(n \geq 1, k \geq 1)$ on '$n(k+1)$' vertices.

Key words and phrases: k-uniform dcsl index, dimension of a poset, lattice.

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INTRODUCTION

Acharya [1] introduced the notion of vertex *set-valuation* as a set-analogue of number valuation. For a graph $G = (V, E)$ and a nonempty set X, Acharya defined a *set-valuation* of G as an injective *set-valued* function $f : V(G) \to 2^X$, and defined a *set-indexer* $f^{\oplus} : E(G) \to 2^X \setminus \{\varnothing\}$ as a *set-valuation* such that the function given by $f^{\oplus}(uv) = f(u) \oplus f(v)$ for every $uv \in E(G)$ is also injective, where 2^X is the set of all subsets of X and '$\oplus$' is the binary operation of taking the symmetric difference of subsets of X.

Acharya and Germina [2], introduced the particular kind of set-valuation for which a metric, especially the cardinality of the symmetric difference, associated with each pair of vertices is k (where k be a constant) times that of the distance between them in the graph [2]. In other words, determine those graphs $G = (V, E)$ that admit an injective set-valued function $f : V(G) \to 2^X$, where 2^X is the power set of a nonempty set X, such that, for each pair of distinct vertices u and v in G, the cardinality of the symmetric difference $f(u) \oplus f(v)$ is k times

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