



# Constructions of 1-Uniform dcsl graphs using Well-graded families of sets

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## Abstract

A 1-uniform dcsl of a graph  $G$  is an injective set assignment function  $f : V(G) \rightarrow 2^X$ ,  $X$  be a non-empty set, such that the corresponding induced function  $f^\oplus : V(G) \times V(G) \rightarrow 2^X \setminus \{\phi\}$  given by  $f^\oplus(uv) = f(u) \oplus f(v)$  satisfies  $|f^\oplus(u, v)| = 1.d(u, v)$  for all distinct  $u, v \in V(G)$ , where  $d(u, v)$  is the length of a shortest path between  $u$  and  $v$ , and  $f(u) \oplus f(v)$  denotes the symmetric difference of the two sets. Let  $\mathcal{F}$  be a family of subsets of a set  $X$ . A *tight path* between two distinct sets  $P$  and  $Q$  (or from  $P$  to  $Q$ ) in  $\mathcal{F}$  is a sequence  $P_0 = P, P_1, P_2, \dots, P_n = Q$  in  $\mathcal{F}$  such that  $d(P, Q) = |P \Delta Q| = n$  and  $d(P_i, P_{i+1}) = 1$  for  $0 \leq i \leq n - 1$ . The family  $\mathcal{F}$  is *well-graded* (or *wg-family*), if there is a *tight path* between any two of its distinct sets. Any family  $\mathcal{F}$  of subsets of  $X$  defines a graph  $G_{\mathcal{F}} = (\mathcal{F}, E_{\mathcal{F}})$ , where  $E_{\mathcal{F}} = \{\{P, Q\} \subseteq \mathcal{F} : |P \Delta Q| = 1\}$ , and we call  $G_{\mathcal{F}}$ , an  $\mathcal{F}$ -induced graph. The purpose of this paper is to examine the existence of 1-uniform dcsl of an induced graph  $G_{\mathcal{F}_1 \cup \mathcal{F}_2 \cup \dots \cup \mathcal{F}_n}$  formed from the finite union of well-graded families  $\mathcal{F}_1, \mathcal{F}_2, \dots$ , and  $\mathcal{F}_n$  by introducing amalgamation techniques in