Malaya Journal of Matematik, Vol. 7, No. 1, 113-117, 2019
https://doi.org/10.26637/MJM0701/0022

On certain associated graphs of set-valued signed graphs

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Abstract
Let X be a non-empty set and let Σ be a signed graph, with corresponding underlying graph G and the signature σ. An injective function $f: V(\Sigma) \to \mathscr{P}(X)$ is said to be a set-labeling of Σ if f is a set-labeling of the underlying graph G and the signature of Σ is defined by $\sigma(uv) = (-1)^{|f(u)\oplus f(v)|}$. A signed graph Σ together with a set-labeling f is known as a set-labeled signed graph and is denoted by Σ_f. In this paper, we discuss the characteristics of certain signed graphs associated with given set-valued signed graphs.

Keywords
Signed graphs, balanced signed graphs, set-valuations of signed graphs.

Mathematics Subject Classification
05C78, 05C10, 05C22.

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Article History: Received 27 June 2018; Accepted 25 January 2019

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1. Introduction

For all terms and definitions, not defined specifically in this paper, we refer to [5, 9, 18] and and for the topics in signed graphs we refer to [19, 20]. Unless mentioned otherwise, all graphs considered here are simple, finite, connected and have no isolated vertices.

Let X be a non-empty set and $\mathscr{P}(X)$ be its power set. A *set-labeling* (or a *set-valuation*) of a graph G is an injective function $f: V(G) \to \mathscr{P}(X)$ such that the induced function $f^{\oplus}: E(G) \to \mathscr{P}(X)$ is defined by $f^{\oplus}(uv) = f(u) \oplus f(v)\ \forall\ uv \in E(G)$, where $\oplus$ is the symmetric difference of two sets. A graph G which admits a set-labeling is called an *set-labeled graph* (or a set-valued graph)(see [1]). A *set-indexer* of a graph G is an injective function $f: V(G) \to \mathscr{P}(X)$ such that the induced function $f^{\oplus}: E(G) \to \mathscr{P}(X)$ is also injective. A graph G which admits a set-indexer is called a *set-indexed graph* (see [1]).

An edge of a graph G having only one end vertex is known as a *half edge* of G and an edge of G without end vertices is called *loose edge* of G.

A *signed graph* (see [19, 20]), denoted by $\Sigma(G,\sigma)$, is a graph $G(V,E)$ together with a function $\sigma: E(G) \to \{+,-\}$ that assigns a sign, either $+$ or $-$, to each ordinary edge in G. The function σ is called the *signature* or *sign function* of Σ, which is defined on all edges except half edges and is required to be positive on free loops. An edge e of a signed graph Σ is said to be a *positive edge* if $\sigma(e) = +$ and an edge $\sigma(e)$ of a signed graph Σ is said to be a *negative edge* if $\sigma(e) = -$. The set E^+ denotes the set of all positive edges in Σ and the set E^- denotes the set of negative edges in Σ. A simple cycle (or path) of a signed graph Σ is said to be *balanced* (see [3, 10]) if the product of signs of its edges is $+$. A signed graph Σ is said to be a *balanced signed graph* if it contains no half edges and all of its simple cycles are balanced. It is to be noted that the number of all negative signed graph is balanced if and only if it is bipartite.

Balance or imbalance is the basic and the most important property of a signed graph. The following theorem, popularly known as *Harary's Balance Theorem*, establishes a criteria for balance in a signed graph.