Vertex (n,k)-choosability of graphs

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Abstract. Let $G = (V, E)$, connected, simple graph of order n and size m and let $V(G) = \{1, 2, ..., n\}$. A graph $G = (V, E)$ is said to be vertex (n,k)-choosable, if there exists a collection of subsets of the vertex set, $\{S_k(v) : v \in V\}$ of cardinality k, such that $S_k(u) \cap S_k(v) = \emptyset$ for all $uv \in E(G)$. This paper initiates a study on vertex (n,k)-choosable graphs and finds the different integer values of k, for which the given graph is vertex (n,k)-choosable.
Keywords: choosability, vertex (n,k)-choosability.

1. Introduction

Throughout this article, unless otherwise mentioned, by a graph we mean a connected, simple graph and any terms which are not mentioned here, the reader may refer to [8]. Let $G = (V, E)$, be a graph of order n and size m, where $V(G) = \{1, 2, ..., n\}$. Given a graph G, a *list assignment* L (or a *list coloring)* of G is a mapping that assigns to every vertex v of G, a finite list $L(v)$ of colors[12]. Also, G is said to be *$\mathcal{L}$-list colorable* if the vertices of G can be properly colored so that each vertex v is colored with a color from $\mathcal{L}(v)$.

Invoking the concept of list-assignments of graphs, the concept of $(a : b)$-choosability was defined and studied in [4].

Definition 1.1. A graph $G = (V, E)$ is $(a : b)$-choosable, if for every family of sets $\{S(v) : v \in V\}$ of cardinality a, there exist subsets $C(v) \subset S(v)$, where $|C(v)| = b$ for every $v \in V$, and $C(u) \cap C(v) = \emptyset$, whenever $u, v \in V$ are adjacent.

The k^{th} choice number of G, denoted by $ch_k(G)$, is the minimum integer n so that G is $(n : k)$-choosable. A graph $G = (V, E)$ is k-choosable if it is $(k : 1)$-choosable. The choice number of G, denoted by $ch(G)$, is equal to $ch_1(G)$. Following this, some interesting studies on choosability of graphs have been done (see [1, 5, 6]).

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