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On α-Vertex Choosability of Graphs

P. Soorya[1] · K. Augusthy Germina[1] · Naduvath Sudev[2]

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Abstract A connected, simple graph G with vertex set $V(G) = \{1, 2, \ldots, n\}$ is said to be vertex (n, k)-choosable, if there exists a collection of subsets $\{S_k(v) \subseteq V(G) : v \in V\}$ of cardinality k, such that $S_k(u) \cap S_k(v) = \emptyset$ for all $uv \in E(G)$, where k is a positive integer less than n. The maximum value of such k is called the vertex choice number of G. In this paper, we introduce the notion of α- choosability of graphs in terms of their vertex (n, k)-choice number and initiate a study on the structural characteristics of α-choosable graphs.

Keywords Independence number · Vertex (n, k)-choosability · Vertex choice number · α-vertex choosability

Mathematics Subject Classification 05C69 · 05C75

For all terms and definitions which are not defined in this paper, we refer to [1, 6, 8]. Unless mentioned otherwise, all graphs we consider in this paper are simple, finite and connected.

✉ Naduvath Sudev
sudev.nk@christuniversity.in

P. Soorya
sooryap2017@gmail.com

K. Augusthy Germina
germinaka@cukerala.ac.in

1 Department of Mathematics, Central University of Kerala, Kasaragod, Kerala, India

2 Department of Mathematics, CHRIST (Deemed to be University), Bengaluru, Karnataka 560029, India

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An *independent set* of a graph G is a set of vertices of G such that no two of them are adjacent. The *independence number* of G is denoted by $\alpha(G)$ and is defined as the cardinality of a maximal independent set in G.

A *list assignment* L (or a *list coloring*) of a graph G is a mapping that assigns to every vertex v of G, a finite list $L(v)$ of colours. Invoking the concept of list-assignments of graphs, the concept of $(a : b)$-choosability was defined and studied in [4] as follows:

Definition 1 (*(a, b)-Choosability of graphs*) [4] A graph $G = (V, E)$ is $(a : b)$-*choosable*, if for every family of sets $\{S(v) : v \in V\}$ of cardinality a, there exist subsets $C(v) \subset S(v)$, where $|C(v)| = b$ for every $v \in V$, and $C(u) \cap C(v) = \emptyset$, whenever $u, v \in V$ are adjacent.

Different variants of this parameter were studied intensively thereafter. Some significant and relevant studies in this area can be seen in [2, 3, 7]. Analogous to the notion of (a, b)-choosability of graphs, the vertex set-oriented (a, b)-choosable graph has been defined in [5] as follows:

Definition 2 (*Vertex (n, k)-choosability of graphs*) [5] A graph $G = (V, E)$ is said to be *vertex (n, k)-choosable*, if there exists a collection of subsets of the vertex set of G, say, $\{S_k(v) : v \in V\}$ of cardinality k, such that $S_k(u) \cap S_k(v) = \emptyset$ for all $uv \in E(G)$.

The maximum value of k so that a graph G is vertex (n, k)-choosable is called the *vertex choice number* of G and is denoted by $\mathcal{V}_{ch}(G)$. The vertex choice number of some fundamental graph classes such as paths, cycles, trees, complete graphs and complete bipartite graphs have been determined in [5].

Not all graphs have their independence number the same as the vertex choice number. For example, the star graph S_n, $n \geq 3$. For S_n, the vertex choice number is $\lfloor \frac{n}{2} \rfloor$, whereas