



New Families of Circulant Graphs Without Cayley Isomorphism Property with $r_i = 2$

V. Vilfred Kamalappan¹

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Abstract

A circulant graph $C_n(R)$ is said to have the *Cayley isomorphism (CI) property* if whenever $C_n(S)$ is isomorphic to $C_n(R)$, there is some $a \in \mathbb{Z}_n^*$ for which $S = aR$. In this paper, we define Type-2 isomorphism, a new type of isomorphism of circulant graphs, different from already known Adam's isomorphism, with $r_i = 2$ and present its properties. At the end of the paper, we present a VB program to show the action of the transformation acting on $C_n(R)$ to obtain Type-2 isomorphism. Type-2 isomorphic circulant graphs have the property that they are isomorphic circulant graphs without *CI-property*.

Keywords Type-1 isomorphic circulant graphs · Type-2 isomorphic circulant graphs · Cayley isomorphism (CI) property · CI-graph · Abelian groups $Ad_n(C_n(R), \circ) = T1_n(C_n(R), \circ), (T2_{n,r}(C_n(R)), \circ), (V_{n,r}(C_n(R)), \circ)$

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Introduction

Beauty comes out of symmetry and asymmetry as well. Investigation of symmetries/asymmetries of structures yield powerful results in Mathematics. Circulant graphs form a class of highly symmetric mathematical (graphical) structures. In 1846 Catalan (cf. [5]) introduced circulant matrices and thereafter properties of circulant graphs have been investigated by many authors [1–25]. Boesch and Tindell [4] studied connectivity of circulant graphs; Elspas and Turner [6] raised a question on the type of isomorphism of $C_{16}(1, 3, 7)$ and $C_{16}(2, 3, 5)$; Sachs [15] studied self-complementary circulant graphs and Alspach, Morris and Vilfred [2,17] settled the conjecture, stated by him, on the existence of self-complementary circulant graphs; Vilfred [18] developed a theory of Cartesian product and factorization of circulant graphs similar to the product and factorization of natural numbers; he also defined a new type of circulant graph isomorphism [19–25] and developed its theory; automorphism of circulant graphs was studied by Li [9], Morris [10], Palfy [14]. In

✉ V. Vilfred Kamalappan
vilfredkamalv@cukerala.ac.in

¹ Department of Mathematics, Central University of Kerala, Tejaswini Hills, Periyar, Kasaragod, Kerala 671 316, India