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FIXED POINT THEOREMS FOR CONDENSING MAPPINGS SATISFYING LERAY-SCHAUDER TYPE CONDITIONS

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ABSTRACT. In this paper, some new fixed point theorems for condensing mappings are established based on a well known result of Petryshyn. We use several Leray-Schauder type conditions to prove new fixed point results. We also obtain generalizations of Altman's theorem and Petryshyn's theorem as well.

1. Introduction and preliminaries

Condensing mappings are arising extensively in differential and integral equations. Hence study of such operators are important part of nonlinear analysis and fixed point theorems related to condensing mappings are very useful in the study of existence of solutions of differential and integral equations.

In 1971, W. V. Petryshyn introduced several fixed point theorems for condensing mapping satisfying Leray-Schauder boundary condition.

Leray-Schauder boundary condition is formulated as follows:

A mapping $T : \overline{B} \to X$ is said to satisfy Leray-Schauder boundary condition if $T(x) \neq \lambda x,\ \forall x \in \partial B,\ \lambda > 1$, where B is the open ball about the origin (see [1]).

The following theorem is due to Petryshyn:

Let B be an open ball about the origin in a general Banach space X. If $T : \overline{B} \to X$ is a condensing mapping (and, in particular, a k-set contraction with $k < 1$) which satisfies the boundary condition $T(x) = \lambda x$ for some x in ∂B, $\lambda \leq 1$, then $F(T)$, the set of fixed points of T in $\overline{B}$, is nonempty and compact.

We need the following preliminary definitions:

Let D be a bounded subset of a metric space X. Define the *measure of noncompactness* $\alpha(D)$ of D by

$$\alpha(D) = \inf\{\varepsilon > 0 : \text{D admits a finite covering of subsets of diameter} < \varepsilon\}.$$

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