



Constructions of 1-Uniform dcsl graphs using Well-graded families of sets

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Abstract

A 1-uniform dcsl of a graph G is an injective set assignment function $f : V(G) \rightarrow 2^X$, X be a non-empty set, such that the corresponding induced function $f^\oplus : V(G) \times V(G) \rightarrow 2^X \setminus \{\phi\}$ given by $f^\oplus(uv) = f(u) \oplus f(v)$ satisfies $|f^\oplus(u, v)| = 1 \cdot d(u, v)$ for all distinct $u, v \in V(G)$, where $d(u, v)$ is the length of a shortest path between u and v , and $f(u) \oplus f(v)$ denotes the symmetric difference of the two sets. Let \mathcal{F} be a family of subsets of a set X . A *tight path* between two distinct sets P and Q (or from P to Q) in \mathcal{F} is a sequence $P_0 = P, P_1, P_2 \dots P_n = Q$ in \mathcal{F} such that $d(P, Q) = |P \Delta Q| = n$ and $d(P_i, P_{i+1}) = 1$ for $0 \leq i \leq n - 1$. The family \mathcal{F} is *well-graded* (or *wg-family*), if there is a *tight path* between any two of its distinct sets. Any family \mathcal{F} of subsets of X defines a graph $G_{\mathcal{F}} = (\mathcal{F}, E_{\mathcal{F}})$, where $E_{\mathcal{F}} = \{\{P, Q\} \subseteq \mathcal{F} : |P \Delta Q| = 1\}$, and we call $G_{\mathcal{F}}$, an \mathcal{F} -induced graph. The purpose of this paper is to examine the existence of 1-uniform dcsl of an induced graph $G_{\mathcal{F}_1 \cup \mathcal{F}_2 \cup \dots \cup \mathcal{F}_n}$ formed from the finite union of well-graded families $\mathcal{F}_1, \mathcal{F}_2, \dots$, and \mathcal{F}_n by introducing amalgamation techniques in