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CHARACTERIZATION OF K-UNIFORM DCSL GRAPHS AND K-GRADED FAMILY OF SETS

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Abstract

A distance compatible set labeling(dcsl) of a connected graph G is an injective set assignment $f: V(G) \rightarrow 2^X$, X being a non empty ground set , such that the corresponding induced function $f^{\oplus} : V(G) \times V(G) \rightarrow 2^X$ given by $f^{\oplus}(u,v) = f(u) \oplus f(v)$ satisfies $|f^{\oplus}(u,v)| = k^f_{(u,v)} d_G(u,v)$ for every pair of distinct vertices $u, v \in V(G)$, where $d_G(u,v)$ denotes the path distance between u and v and $k^f_{(u,v)}$ is a constant, not necessarily an integer, depending on the pair of vertices u, v chosen. A dcsl f of G is k -uniform if all the constant of proportionality with respect to f are equal to k, and if G admits such a dcsl then G is called a k -uniform dcsl graph. Let $\mathcal{F}$ be a family of subsets of a set X. A k -tight path between two distinct sets P and Q (or from P to Q) in $\mathcal{F}$ is a sequence $P_0 = P, P_1, P_2 \ldots P_n = Q$ in $\mathcal{F}$ such that $d(P,Q) = |P \oplus Q| = kn$ and $d(P_{1i}, P_{i+1}) = k$ for $0 \leq i \leq n-1$. The family $\mathcal{F}$ is k -graded family, if there is a k -tight path between any two of its distinct sets. In this paper, we establish the relationship between k -uniform dcsl graphs and k -graded family of sets.

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Key Words: dcsl graphs, k -uniform dcsl graphs, k -graded family of sets, $\mathcal{F}_k$ -induced graph.

1 Introduction

Acharya [1] introduced the notion of vertex set valuation as a set analogue of number valuation. For a graph $G = (V, E)$ and a non empty set X, Acharya defined a set valuation of G as an injective set valued function $f: V(G) \rightarrow 2^X$, and he defined a set-indexer as a set valuation such that the function $f^{\oplus} : E(G) \rightarrow 2^X \setminus \{\varphi\}$ given by $f^{\oplus}(uv) = f(u) \oplus f(v)$ for every $uv \in E(G)$ is also injective, where 2^X is the set of all the subsets of X and $\oplus$ is the binary operation of taking the symmetric difference of subsets of X.

Acharya and Germina, who has been studying topological set valuation[7], introduced the particular kind of set valuation for which a metric, especially the cardinality of the symmetric