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## FG- COUPLED FIXED POINT THEOREMS IN CONE METRIC SPACES

The concept of  $FG$ - coupled fixed point introduced recently is a generalization of coupled fixed point introduced by Guo and Lakshmikantham. A point  $(x, y) \in X \times X$  is said to be a coupled fixed point of the mapping  $F : X \times X \rightarrow X$  if  $F(x, y) = x$  and  $F(y, x) = y$ , where  $X$  is a non empty set. In this paper, we introduce  $FG$ - coupled fixed point in cone metric spaces for the mappings  $F : X \times Y \rightarrow X$  and  $G : Y \times X \rightarrow Y$  and establish some  $FG$ - coupled fixed point theorems for various mappings such as contraction type mappings, Kannan type mappings and Chatterjea type mappings. All the theorems assure the uniqueness of  $FG$ - coupled fixed point. Our results generalize several results in literature, mainly the coupled fixed point theorems established by Sabetghadam et al. for various contraction type mappings. An example is provided to substantiate the main theorem.

*Key words and phrases:*  $FG$ - coupled fixed point, cone metric space, contraction type mappings.

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### 1 INTRODUCTION

The classical Banach contraction theorem is proved to be one of the most fruitful and durable results in metric fixed point theory. Due to its enormous applications, several authors have studied and made very many generalizations of Banach contraction principle. In 2004 A. C. M. Ran and M. C. B. Reurings [1] proved an analogue of Banach contraction principle in partially ordered metric spaces and used the theorem to solve matrix equations. Following this, J. J. Nieto and R. R. Lopez [5, 6] established several fixed point theorems in partially ordered metric spaces and obtained applications to periodic boundary value problems. As an extension of fixed point, a new concept called coupled fixed point is introduced by Guo and Lakshmikantham [2]. They investigated some coupled fixed point theorems of mixed monotone operator, and applied their results to solve initial value problem of ordinary differential equations with discontinuous right hand sides. Using the notion of coupled fixed points they explored the existence and uniqueness of fixed point of non-monotone operator. Later Gnana Bhaskar and Lakshmikantham [13] established existence and uniqueness theorems of coupled fixed point for mixed monotone mappings defined on partially ordered complete metric spaces satisfying contraction type condition and applied their result to solve periodic boundary value problems. After the work of Gnana Bhaskar and Lakshmikantham, in 2009 Ciric and Lakshmikantham [14] introduced a new mapping called mixed  $g$ - monotone mapping. Using this,

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