FG- COUPLED FIXED POINT THEOREMS IN GENERALIZED METRIC SPACES

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Abstract: In this paper we establish FG- coupled fixed point theorems in partially ordered complete S^* metric space. We illustrate our results with examples. An S^* metric is an n-tuple metric from n-product of a set to the non negative reals. Our theorem generalizes the main results of Gnana Bhaskar and Lakshmikantham [T. Gnana Bhaskar, V. Lakshmikantham; Fixed point theorems in partially ordered metric spaces and applications; Nonlinear Analysis 65 (2006) 1379 - 1393].

Keywords: FG- Coupled Fixed Point, Mixed Monotone Property, Partially Ordered Set, S^* Metric.

Introduction: A metric on a set is a function from the bi-product of the set to the non negative reals satisfying certain axioms. Many authors have introduced different types of metrics by making some changes in the axioms or in the domain or co- domain [3], [4], [8], [11], [15], [16]. Followed by this, they have proved several fixed point results under such metric spaces. Now a lot of fixed point and coupled fixed point results are available under different types of metric spaces [2], [6], [7], [9], [13], [14]. An n-tuple metric called S^* metric is the latest development in this direction. In [1] Abdellaoui, M.A. and Dahmani, Z. introduced this concept and they have proved fixed point results in S^*metric spaces. But the same concept can be seen in [10], under a different name. In [10] Mujahid Abbas, Bashir Ali and Yusuf I Suleiman coined the name A- metric to this concept and they have proved common coupled fixed point theorems with an illustrating example. Throughout this paper we will use the notation $S^*$to indicate this concept of n-tuple metric.

In this paper we use the concept of S^*metric to obtain certain FG- coupled fixed point results. Our work generalizes the results of Gnana Bhaskar and Lakshmikantham [5]. The useful definitions and results to prove the main theorems follows:

Definition 1[1], [10]: An S^* metric on a nonempty set X is a function $S^*: X^n \to [0,\infty)$ satisfying:
[i] $S^*(x_1,x_2,\dots,x_n) \geq 0$,
[ii] $S^*(x_1,x_2,\dots,x_n) = 0$if and only if $x_1 = x_2 = \cdots = x_n$,
[iii] $S^*(x_1,x_2,\dots,x_n) \leq S^*(x_1,\dots,x_1,a) + S^*(x_2,\dots,x_2,a) + \cdots + S^*(x_n,\dots,x_n,a)$ for any $x_1,x_2,\dots,x_n \in X$. The pair (X,S^*) is called S^* metric space.

Lemma 1[1], [10]: Suppose that (X,S^*) is an S^* metric space. Then for all $x_1,x_2 \in X$, we have $S^*(x_1,x_1,\dots,x_1,x_2) = S^*(x_2,x_2,\dots,x_2,x_1)$

Definition 2[1], [10]: We say that the sequence $\{x_p\}_{p\in N}$ of the space X is convergent to x if $S^*(x_p,x_p,\dots,x_p,x) \to 0$ as $p \to \infty$. We write $\lim_{p\to\infty} x_p = x$

Definition 3[1], [10]: We say that the sequence $\{x_p\}_{p\in N}$ of the space X is of Cauchy if for each $\epsilon > 0$, there exist $p_0 \in N$ such that for any $p,q \geq p_0$, $S^*(x_p,x_p,\dots,x_p,x_q) < \epsilon$.

The space (X,S^*) is complete if all its Cauchy sequences are convergent.

Lemma 2[1], [10]: Let (X,S^*) be an S^* metric space. If $\{x_p\}_{p\in N}$ in X converges to x, then x is unique.

Definition 4[12]: Let $(X,\leq_{p_1})$ and $(Y,\leq_{p_2})$ be two partially ordered sets and $F: X\times Y \to X$ and $G: Y\times X \to Y$ be two mappings. An element $(x,y) \in X\times Y$ is said to be an FG- coupled fixed point if $F(x,y) = x$ and $G(y,x) = y$.