

Coupled fixed point theorems in Partially ordered sets

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Abstract

Coupled fixed point theorems and coupled coincidence theorems for both mixed monotone mapping and mixed g -monotone mapping are proved in partially ordered sets.

Keywords: Partially ordered set, Coupled fixed point; Mixed monotone property; Mixed g - monotone property.

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1 Introduction

Fixed point theory is one of the most important area of research in Mathematics. Tarski's theorem, related to the existence of fixed point for an order preserving mapping defined on a complete lattice is one of the well known result in fixed point theory. In 1961 S. Abian and A. B. Brown [1] established fixed point theorems in partially ordered set which extends Tarski's fixed point theorem to a more general setting. Followed by this several authors have proved extensions and generalizations of fixed point theorems in partially ordered sets [3, 4, 6, 9, 11].

In 1987 Guo and Lakshmikantham [8] introduced the concept of coupled fixed point. Later Gnana Bhaskar and Lakshmikantham [7] demonstrated several coupled fixed point theorems for mixed monotone mappings defined on partially ordered complete metric spaces. In [12] Ciric and Lakshmikantham introduced a mapping called mixed g -monotone mapping, a generalization of mixed monotone mapping and proved coupled coincidence theorem for a self mapping g and a mixed g - monotone mapping defined on a partially ordered complete metric space. In [2, 5, 10, 13, 14, 15] many more coupled fixed point theorems are proved.

In this paper three coupled fixed point theorem and two coupled coincidence theorems in partially ordered sets are proved. Some needful definitions are given below:

Definition 1.1. [7] Let X be a set and $f : X \times X \rightarrow X$ be a mapping then a point $(x, y) \in X \times X$ is said to be a coupled fixed point if $f(x, y) = x$ and $f(y, x) = y$.

Definition 1.2. [7] Let (X, \leq) be a partially ordered set and let $f : X \times X \rightarrow X$ be a mapping then f is said to satisfy mixed monotone property if for all $x, y, u, v \in X$, $x \leq u$ implies that $f(x, y) \leq f(u, y)$; and $v \leq y$ implies that $f(x, y) \leq f(x, v)$

Definition 1.3. [12] Let (X, \leq) be a partially ordered set. Let $f : X \times X \rightarrow X$ and $g : X \rightarrow X$ be two mappings then f is said to satisfy mixed g - monotone property if for all $x, y, u, v \in X$, $g(x) \leq g(u)$ implies that $f(x, y) \leq f(u, y)$; and $g(v) \leq g(y)$ implies that $f(x, y) \leq f(x, v)$

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