FG-COUPLED FIXED POINT THEOREMS FOR VARIOUS CONTRACTIONS IN PARTIALLY ORDERED METRIC SPACES

PRAJISHA E. AND SHAINI P.

ABSTRACT. In this paper we introduce FG-coupled fixed point, which is a generalization of coupled fixed point for nonlinear mappings in partially ordered complete metric spaces. We discuss existence and uniqueness theorems of FG-coupled fixed points for different contractive mappings. Our theorems generalizes the results of Gnana Bhaskar and Lakshmikantham [1].

1. INTRODUCTION

Fixed point theory has many applications in nonlinear analysis. In [3–5] the authors presented fixed point theorems in partially ordered metric spaces and their applications. As a generalization of fixed points, in [2] Guo and Lakshmikantham introduced the concept of abstract coupled fixed points for some operators, thereafter Gnana Bhaskar and Lakshmikantham in [1] introduced coupled fixed points and mixed monotone property for contractive mappings on partially ordered metric spaces. They proved interesting coupled fixed point results in [1]. An interesting application of their result is that it can be used to find the solution of periodic boundary value problem, moreover it guarantees the uniqueness of the solution. Followed by this, several authors established new coupled fixed point theorems in partially ordered complete metric spaces and in cone metric spaces. In [6] Sabetghadam, Masiha and Sanatpour proved generalization of results of Gnana Bhaskar and Lakshmikantham in cone metric spaces.

In this paper we introduce a new concept which is a generalization of coupled fixed point and prove existence theorems for contractive mappings in partially ordered metric spaces. Some examples are also discussed to illustrate our results. We recall the basic definitions.

Definition 1.1 ([1]). Let (X, \leq) be a partially ordered set and $F: X \times X \to X$. We say that F has the mixed monotone property if F(x,y) is monotone non decreasing

²⁰¹⁰ Mathematics Subject Classification. 47H10, 54F05.

Key words and phrases. FG-coupled fixed point; Coupled fixed point; Mixed monotone property; Partially ordered set.