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Set of periods of a subshift

K ALI AKBAR[1,*] and V KANNAN[2]

[1]Department of Mathematics, Central University of Kerala, Kasaragod 671 316, India
[2]School of Mathematics and Statistics, University of Hyderabad, Hyderabad 500 046, India
*Corresponding author.
E-mail: aliakbar.pkd@gmail.com; vksm@uohyd.ernet.in

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Abstract. In this article, subsets of $\mathbb{N}$ that can arise as sets of periods of the following subshifts are characterized: (i) subshifts of finite type, (ii) transitive subshifts of finite type, (iii) sofic shifts, (iv) transitive sofic shifts, and (v) arbitrary subshifts.

Keywords. Subshift of finite type; transitive subshift; strongly connected digraph; sofic shift.

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1. Introduction and preliminaries

The subshifts of finite type (SFT) form a very important class of dynamical systems. There are plenty of books that explain how their study would throw light on still larger classes of dynamical systems (see [7,8,11]).

A dynamical system is a pair (X, f), where X is a metric space and f is a continuous self map. For each dynamical system (X, f), the set $\text{Per}(f) = \{n \in \mathbb{N} : \exists\, x \in X$ such that $f^n(x) = x \neq f^m(x)\ \forall m < n\}$ consisting of the lengths of the cycles, is a subset of the set $\mathbb{N}$ of positive integers. Given a class of dynamical systems, we obtain in this way a family of subsets of $\mathbb{N}$. There have been attempts to describe explicitly this family of subsets of $\mathbb{N}$, for various classes of dynamical systems (see [2,3,5,6,8,9,12] and [13]).

It is therefore natural to ask for a description of sets of periods for the class of all subshifts of finite type. In this paper, we answer this and two other related questions (see Theorems 1 and 3). Here, we concentrate on two-sided shifts. The case of one-sided shifts is similar.

Let $\mathcal{A}$ be a non-empty finite set, called alphabet with discrete topology and consider the set $\mathcal{A}^{\mathbb{Z}}$, which denotes the set of doubly-infinite sequences $(x_i)_{i\in\mathbb{Z}}$ where each $x_i \in \mathcal{A}$, with product topology. It is compact and metrizable. The shift is the homeomorphism $\sigma : \mathcal{A}^{\mathbb{Z}} \to \mathcal{A}^{\mathbb{Z}}$ given by $\sigma(x)_i = x_{i+1}$ for all $i \in \mathbb{Z}$. The pair $(\mathcal{A}^{\mathbb{Z}}, \sigma)$ forms a dynamical system called a full shift. We say that a point $x \in \mathcal{A}^{\mathbb{Z}}$ is periodic if $\sigma^n x = x$ for some $n \in \mathbb{N}$, where σ^n is the composition of σ with itself n times. The smallest such positive integer n is called the σ-period (simply period) of x. A subset $A \subset \mathcal{A}^{\mathbb{Z}}$ is called σ-invariant

