

## Simple dynamical systems

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### ABSTRACT

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*In this paper, we study the class of simple systems on  $\mathbb{R}$  induced by homeomorphisms having finitely many non-ordinary points. We characterize the family of homeomorphisms on  $\mathbb{R}$  having finitely many non-ordinary points upto (order) conjugacy. For  $x, y \in \mathbb{R}$ , we say  $x \sim y$  on a dynamical system  $(\mathbb{R}, f)$  if  $x$  and  $y$  have same dynamical properties, which is an equivalence relation. Said precisely,  $x \sim y$  if there exists an increasing homeomorphism  $h : \mathbb{R} \rightarrow \mathbb{R}$  such that  $h \circ f = f \circ h$  and  $h(x) = y$ . An element  $x \in \mathbb{R}$  is ordinary in  $(\mathbb{R}, f)$  if its equivalence class  $[x]$  is a neighbourhood of it.*

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### 1. INTRODUCTION

A dynamical system is a pair  $(X, f)$  where  $X$  is a metric space and  $f$  is a continuous self map on  $X$ . Two dynamical systems  $(X, f)$ ,  $(Y, g)$  are said to be topological conjugate if there exists a homeomorphism  $h : X \rightarrow Y$  (called topological conjugacy) such that  $h \circ f = g \circ h$ . The properties of dynamical systems which are preserved by topological conjugacies are called dynamical properties.