Applied General Topology

Appl. Gen. Topol. 21, no. 2 (2020), 215-233
doi:10.4995/agt.2020.12929
© AGT, UPV, 2020

The class of simple dynamical systems

K. Ali Akbar

Department of Mathematics, Central University of Kerala, Kasaragod - 671320, Kerala, India. (aliakbar.pkd@gmail.com, aliakbar@cukerala.ac.in)

Communicated by F. Balibrea

Abstract

In this paper, we study the class of simple dynamical systems on $\mathbb{R}$ induced by continuous maps having finitely many non-ordinary points. We characterize this class using labeled digraphs and dynamically independent sets. In fact, we classify dynamical systems up to their number of non-ordinary points. In particular, we discuss about the class of continuous maps having unique non-ordinary point, and the class of continuous maps having exactly two non-ordinary points.

2010 MSC: *54H20; 26A21; 26A48.*

Keywords: *special points; non-ordinary points; critical points; order conjugacy; order isomorphism; labeled digraph; dynamically independent set.*

1. Introduction

A dynamical system is a pair (X, f), where X is a metric space and f is a continuous self map on X. Two dynamical systems (X, f) and (Y, g) are said to be *topologically conjugate* (or simply *conjugate*) if there exists a homeomorphism $h : X \to Y$ (called topological conjugacy) such that $h \circ f = g \circ h$. We simply say that f is conjugate to g, and we write it as $f \sim g$. In the case when h happens to be an increasing homeomorphism (for example, when $X = \mathbb{R}$ or an interval) we say that f and g are *increasingly conjugate* or *order conjugate*. When we are working with a single system, any self conjugacy can utmost shuffle points with same dynamical behavior. Therefore a point which is unique up to its behavior must be fixed by every self conjugacy. On the

Received 02 January 2020 – Accepted 17 May 2020