Research Article

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Identification of an unknown coefficient in KdV equation from final time measurement

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Abstract: In this article, we study an inverse problem of reconstructing a space dependent coefficient in a generalized Korteweg–de Vries (KdV) equation arising in physical systems with variable topography from final time overdetermination data. First the identification problem is transformed into an optimization problem by using optimal control framework and existence of a minimizer for the cost functional is established. Then we prove a stability estimate for retrieving the unknown coefficient in KdV equation with the upper bound of given measurements. The local uniqueness of the coefficient is also discussed.

Keywords: KdV equation, Inverse problem, stability, optimal control

MSC 2010: 35Q53, 45Q05, 49J20

1 Introduction

Wave motion is one of the broadest scientific subjects and it can be studied at different technical level depend upon the medium of presence in nature. A special kind of wave motion is the one-dimensional propagation of long water waves in channels of relative shallow depth and flat bottom that have been modelled by Korteweg and de Vries [21] as the following nonlinear evolution equation

$$u_t(x,t) + u_x(x,t) + u_{xxx}(x,t) + u(x,t)u_x(x,t) = 0.$$

However, since these waves can occur in regions of variable bottom topography, where there is a need to take into account of the variation of the background medium, Johnson [16, 17] and Grimshaw [10, 11] proposed a governing equation for weakly nonlinear unidirectional long waves over nonflat bottom channels given by the variable coefficient KdV equation as follows:

$$u_t(x,t) + (c(x)u(x,t))_x + c^2(x)h(x)u_{xxx}(x,t) + \frac{c(x)}{h(x)}u(x,t)u_x(x,t) = 0,$$

where $u(x,t)$ is the evolution of surface elevation above the undisturbed water depth $h(x)$ at position x at time t and $c(x)$ is the linear long wave speed. The first two terms are the dominant terms and by themselves describe the propagation of linear long wave with speed $c(x)$. The remaining terms represent, respectively, weak linear dispersion and weak nonlinear effects. There are other nonlinear generalization of KdV equation (see [5]) and certain corrections of the coefficients have also been proposed to study the problems with long waves over highly variable topography (see [15]).

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