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MATHEMATICAL GAME THEORY AND APPLICATIONS

An $M/M/1$ Queueing-Inventory System with Working Vacations, Vacation Interruptions and Lost Sales

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Abstract—We consider a single server queueing-inventory system under (s, Q) replenishment strategy with working vacations, vacation interruptions and lost sales. The server provides service at a lower rate during working vacations than while in normal mode of service. If a working vacation realizes while providing service in that mode, then the server continues in the present status until the current service is completed. Upon that it switches to the normal mode of service, provided there is at least one customer waiting. If no customer is waiting at this point of time, it goes for vacation. We also assume that if there are customers in the system at a service completion epoch during a working vacation, the server will come back to the normal working mode, otherwise the server will stay in the working vacation mode. With the system having infinite capacity, the stability condition for the system is obtained, followed by computation of the steady-state probability vector and discussion. Various performance measures are evaluated. In addition, the busy period analysis is provided and the stationary waiting time distribution in the queue is derived. Numerical illustrations are provided to illustrate the system performance, and an optimization problem is also discussed.

Keywords: queueing-inventory, positive service time, server vacation, lost sales

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1. INTRODUCTION

Until the early nineties of the last century, it was assumed that inventory service time is negligible. Nevertheless, this is too unrealistic an assumption. The first reported works on inventory with positive service time are by Melikov and Molchanov [15] and Sigman and Simchi-Levi [26]. Thereafter, only towards the beginning of this century did researchers start further investigation in this direction. Noted among these are Schwarz et al. ([22, 23]); Baek and Moon ([1, 2]), Baek et al. [3]; Safari et al. [21] (arbitrarily distributed lead time); Krishnamoorthy et al. ([8, 9]); Krenzler and Daduna [7] and Shajin et al. [24] among a few others. In a very recent contribution, Shajin and Krishnamoorthy [25] consider a queueing-inventory model with retrials of customers; wherein they obtain product form solution for the steady-state probability distribution. Another recent contribution of interest is by Melikov and Shahmaliyev [17], on a single server queueing system with perishable inventory and retrials of customers from an orbit with infinite capacity; they discuss the cost minimisation problem of choosing the critical level of inventory. A survey on queueing-inventory literature covering developments up to 2009 can be found in Krishnamoorthy et al. [10]. An exhaustive survey of developments is provided by Krishnamoorthy et al. [11].